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The Common Core in Mathematics at Nine Years: An Analysis

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Opinions are those of the author.

This paper is exclusively about the Common Core Standards for School Mathematics, the CCSSM. But it is not about the politics of the Common Core, nor is it about the substance of the Common Core – not about how fractions or geometry or statistics or early number are treated, not about the standards for mathematical practice, not about the timing of teaching various ideas.

These remarks deal with one question: Is the Common Core meeting the goals, or on its way to meeting the goals for which it was created nine years ago?

We begin with a basic question: What do we mean by the Common Core? The simple answer is that the Common Core refers to the Common Core State Standards themselves, announced in a document that first appeared nine years ago, in 2010, prepared by a committee under the auspices of the National Governors' Association and the Council of Chief State School Officers.

The original online document, which to my knowledge has not been changed, consisted of a 3-page introduction, 3 pages describing 8 standards for mathematical practice, 76 pages detailing standards for mathematical content at each of the grades K-8 and high school, a 6-page glossary, and a bibliography. Its current form can be found at www.corestandards.org/Math/.

The main goal of the Common Core is found in the first sentence of the original document following a few quotes: "For over a decade, research studies of mathematics education in high-performing countries have pointed to the conclusion that the mathematics curriculum in the United States must become substantially more focused and coherent in order *to improve mathematics achievement in this country*." (emphasis mine) The document and all else associated with the Common Core are means to this end.

An appendix to the original Common Core document, "Designing high school mathematics courses based on the Common Core State Standards", appeared in 2010 almost immediately after the appearance of the standards themselves. That appendix describes four model course pathways to achieve the standards, a traditional algebra-geometry-algebra pathway, an integrated pathway, and compacted versions of each pathway that would allow students to reach calculus or other college-level courses by their senior year. It's a detailed appendix, 148 pages in length, and can be seen at www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf.

The appendix has not received much attention despite its detail, or perhaps because of its detail, except that it has moved some places to adopt the integrated pathway or to provide that pathway as an alternative to the traditional sequence. Yet the amount of detail given to grades 9-12 reflects that the goal of the Common Core is specifically the improvement of the preparation of students for college-level mathematics. That is, despite the common phrase that the Common Core is designed to improve the preparation of students for college *and career* readiness, there seems to be a tacit assumption that the best way to prepare students for careers is to improve their preparation for college mathematics courses, and specifically for college-level calculus and statistics.

The framers of the Common Core have asserted that the CCSSM is not a curriculum (http://www.corestandards.org/about-the-standards/myths-vs-facts/), but the CCSSM contain the three main qualities of a curriculum: scope, sequence, and timing. In an early TIMSS report, three types of curriculum are identified, one of which describes the Common Core: "What the students know and can do (the attained curriculum) needs to be compared to what teachers report teaching (the implemented curriculum) and to the content that is called for in official policies (the intended curriculum)." (Robitaille , 1997) To people who study curriculum the CCSSM constitute what is called the intended or ideal curriculum.

The TIMSS researchers acknowledge that this three-faceted view of curriculum comes from the Second International Study - SIMS, which was done in the early 1980s. And the SIMS framework can be traced to a more complicated five-faceted framework developed by John Goodlad and his students at UCLA in the late 1970s in a large project titled "A Study of Schooling" (Goodlad, Sirotnik, and Overman, 1979).

Obviously the goal is that the achieved curriculum be as close as possible to the intended or ideal curriculum. The inherent problem in all of standard-setting is that there is slippage from the intended curriculum to the implemented curriculum. Classes tend not to cover everything that is in the intended curriculum. There is further slippage from the implemented curriculum to the achieved curriculum because, except for the very best students, students do not learn everything that was taught.

To minimize this slippage, it is helpful to speak of two more kinds of curriculum. The textbook or materials curriculum is the curriculum found in the materials for learning. The tested curriculum is the curriculum as found in the tests given to students. (See Figure 1.)

The policy makers who created the Common Core dealt with the slippage from the implemented to the achieved curriculum by funding the consortia PARCC and Smarter Balanced with $360,000,000 to create tests that would cover the Common Core and nothing else. The Common Core creators themselves worked to minimize the slippage from the intended to the implemented curriculum by almost immediately creating a K-8 Publishers' Criteria for the Common Core Standards for Mathematics (www.corestandards.org/Assets/Math_Publishers_Criteria_K-8_Summer 2012_FINAL.pdf). A few years later the organization Ed Reports was established to evaluate published materials for their adherence to the Common Core (www.edreports.org).

Figure 1. The many kinds of curriculum.

The **intended (ideal) curriculum**, as set down in documents like the NCTM Standards and the CCSSM.
The **textbook (materials) curriculum** is the curriculum found in the materials for learning.
The **implemented (taught) curriculum** is the curriculum that is taught, the one experienced by students.
The **tested (examined) curriculum** is the curriculum as found in the tests given to students.
The **achieved (attained) curriculum** is the curriculum that is learned by students.

The result has been a massive structure to align materials, teaching, and tests to the Common Core standards in an attempt to improve mathematics performance. And for many people, this full framework is what we mean by the Common Core. And this framework is in existence even in many states that have modified the CCSSM and assert that they are not Common Core states.

Of course there still can be slippage, particularly from the materials curriculum to the implemented curriculum. Teachers have to be willing to cover the material and understand the mathematics and the pedagogy well enough to implement not only the content of the Common Core but also its spirit. NCSM and NCTM seem to understand the need for teachers to be able to deal with the pedagogy – emphasizing the Standards for Mathematical Practice, but the mathematical *knowledge* required for teaching the aspects of the Common Core has been given less attention from the publications of both organizations. This in my opinion is particularly relevant in a national climate where so many new teachers have not gone through full 4-year teacher preparation programs.

But the structure has been so successfully established that there has been for the past few years no major curriculum development work in the United States except to modify programs to adhere to the Common Core. The downside is that the U.S., which was a world leader in curriculum development research just 20 years ago, is now not even a bit player.

However, the overall picture is that the Common Core creators, from policy makers to the writers of the standards, developed a structure that, for the United States, is an unprecedented attempt to implement, from the top down, a curriculum with minimal slippage from its objectives to its manifestations in the classroom.

To What Extent Has the Common Core Been Implemented? - Content

As we all know, the Common Core standards were voluntary standards and were not accepted everywhere. Five states immediately rejected them (populations in millions): Texas (28.3), Virginia (8.5), Minnesota (5.6), Nebraska (1.9), and Alaska (0.7), with a total population of about 44 million, that is, with about 13.5% of the nation's population and I assume about the same percent of the student population. It is interesting that Minnesota is unique among all states; it has gone with the Language Arts Common Core standards but rejected those in mathematics.

Three other states immediately announced that they were leaving the Common Core: Oklahoma, South Carolina, and Indiana. Oklahoma created standards quite different from the Common Core so can be added to the original group. But the standards of South Carolina and Indiana differ so little from the CCSSM that the move to "leave the common core" means basically that the state was asserting its constitutional right to legislate education in its own state and would not be bound by what was seen as a national curriculum. Adding Oklahoma means about 15% of U.S. students are not in a state that follows the CCSSM either strictly or very closely.

Other states that announced later that they have a major rewrite or replacement of the Common Core are at least the following: Arizona (7.0), Louisiana (4.7), Arkansas (3.0), Missouri (6.1), Tennessee (6.7), West Virginia (1.8), New Jersey (9.0), and New York (19.8). These states have 39% of the U.S. population. Earlier this year Florida (21.0) announced a commission to create new standards by next year. But these departures have not changed the fundamental design aspects of the Common Core movement, namely to have a test at each grade 3 through 8 and in high school over specific objectives and a curriculum that meets those objectives.

To What Extent Has the Common Core Been Implemented? - Tests

The situation with regard to *tests* over the Common Core is more complex than the situation with regard to objectives and has changed significantly even since last year. In Table 1, created from a report in Education Week in March, I list the creators of the tests. I classify a state's tests as "state-specific" if the tests used in their state are created especially for the state, though some outside organizations may be being employed to create tests for more than one state. A number of states are using combinations of tests or give choices, particularly at the high school level.

Table 1. Creators of state tests used in 2018-19

Test creator	States using in Gr 3-8	States using in Gr 9-12
SBAC (Smarter Balanced)	CA, CT, DE, HI, ID, MT, NV, OR, SD, VT, WA	CA, HI, ID, OR, SD, VT, WA
PARCC	DC, IL, MD, NJ, NM	DC, MD, NJ, NM

State specific	AL, AK, AZ, CO, FL, GA, IN, IA, KS, KY, LA, ME, MA, MI, MN, MS, MO, NB, NH, NY, NC, ND, OH, OK, PA, RI, SC, TN, TX, UT, VA, WI, WV, WY	AK, AZ, FL, GA, IN, IA, KS, KY, LA, MA, MN, MS, MO, NY, NC, ND, OH, PA, SC, TN, TX, UT, VA, WY
ACT	AR	AL, AR, HI, ID, KY, MI, MS, MT, NB, NV, NC, OH, OK, TN, UT, WI, WY
SAT	MI	CO, CT, DE, DC, ID, IL, ME, MI, NH, OH, OK, RI, TN, WV

Source: Education Week, March 14, 2019. For comparison with 2017-18, see the Education Commission of the States 50-State Comparison at http://ecs.force.com/mbdata/mbquestrt?rep=SUM1802.

From this table, we see that PARCC and Smarter Balanced do not control Common Core test creation as was expected by the U.S. Congress when it funded those two consortia. This year, in grades 3-8, only 11 states remain using Smarter Balanced tests and 4 states and DC remain using part or all of PARCC tests, with a total of about 37% of the nation's population. At grades 9-12, the dominant forces are ACT and SAT, and with more students tested using the SAT than are tested by Smarter Balanced and PARCC combined. So, in all, the notion of having one or two tests nationwide specifically over the Common Core has disappeared. Indeed, you might say that in testing, the situation is not that much different than it was a generation ago when at the elementary level a handful of standardized tests dominated the market: The Iowa Test of Basic Skills, the California Achievement Test, the Metropolitan Achievement Test, the Stanford Achievement Test, and the Comprehensive Test of Basic Skills. And at the high school level, there is less in common in the current tests than there was when only the ACT and SAT and the NY state regents were used as barometers of performance.

The fact that many different tests are now being used to evaluate student performance on a Common Core curriculum does not mean that the Common Core has not been a success. Having everyone use the same test or same two tests was not a goal of the Common Core. The alignment of the intended curriculum and the tested curriculum was part of the *means* to achieving the goal which was and remains, as we said earlier, to improve student performance in mathematics, and in all these states at least at grades 3-8, the tests are aligned with the state intended curriculum.

Is It Too Early to Make a Judgment Regarding the Common Core?

Having more tests makes it more difficult to answer the broader question of the impact of the Common Core on performance. But the major difficulty in having all these tests is that only a small minority of states are using the same tests now that they used when they first were implementing the Common Core.

Nine years since the CCSSM first appeared may be too early to give a definitive answer. Even the most optimistic schedule for implementation would suggest that there should not be any effects of the Common Core for a few years because it takes time for any development to make it into the system. First, it takes time to write materials. Second, materials must be adopted – and adoption schedules are often set independently of any suggested changes in curriculum and adoptions tend to be from 5 to 7 or more years apart. Third, the changes have to be accepted by teachers, often requiring some professional development. And fourth, students have to learn to adjust to the materials, a process that typically takes more than a school year.

Indeed, since the Common Core standards start at kindergarten and the testing does not begin until grade 3, it is easy to argue that a minimum of three years after implementation would be needed to attribute any changes in performance in mathematics to the Common Core, and then only at grade 3. If implementation at 1st grade was done within 3 years, say for the 2013-14 school year, those students would this year be only in grade 6 and we would have results only up to grade 5. It would take until the 2023-2024 school year for any student to have gone through the full Common Core curriculum for grades 1 to 11.

Few people are willing to wait that long for results. But after 9 years, we ought to be able to have some idea regarding whether the Common Core has been a success or not.

To judge improvement, we need to have test score base lines and multi-year or longitudinal studies from which to judge.

One line of thought is to go back to 2010 to see how students were performing then, and compare performance then with recent performance. Another line of thought is that we should take as the base line the performance of the first students to take the tests over the CCSSM, perhaps in the school years 2013-14 or 2014-15, when the PARCC and Smarter Balanced tests were first given in many states. Which year you wish to take as a base line may depend on the extent to which you think the Common Core objectives are different from objectives of curricula used in 2010. How far you wish to go back depends on the test to be examined and the available data.

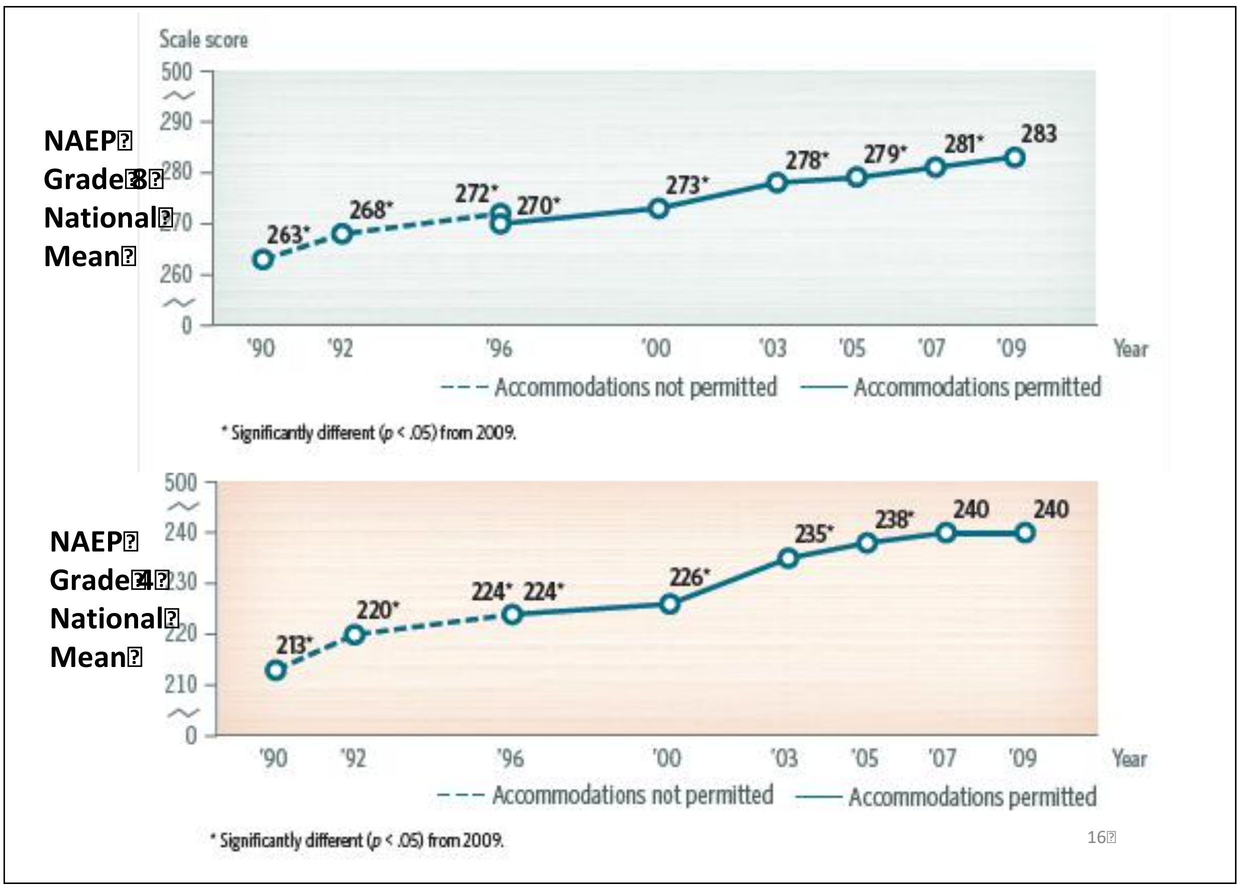
Many tests are possible candidates: NAEP, the SAT and ACT, PARCC and SBAC, other state tests, studies from individual colleges or universities, and studies in individual school districts.

NAEP Results before the CCSSM

The gold standard of longitudinal studies is NAEP, the National Assessment of Educational Progress. NAEP tests, given at grades 4, 8, and 12 undergo a multi-year process from framework to use and are the largest and most carefully analyzed tests in the U.S. that attempt to provide a picture of the entire U.S. school population. Furthermore, recent NAEP tests have been administered every two years in the odd-numbered years and provide state-by-state results, so it is possible to compare performance in Common Core states with those states that did not go with the Common Core.

If we are trying to compare over years, one basis of comparison is the national mean score. We begin with the NAEP mean scores at the time of the writing of the Common Core standards. Since the NCTM Standards of 1989, there had been a vast improvement in national mean performance in mathematics, as shown in Figure 2. At 4th grade, the mean score had gone from 213 to 240 but had not changed much since 2005; at 8th grade, the mean score had increased rather steadily from 263 to 283. The total increase at each grade was generally felt to indicate that students in 2009 were one grade ahead of where students were in 1990.

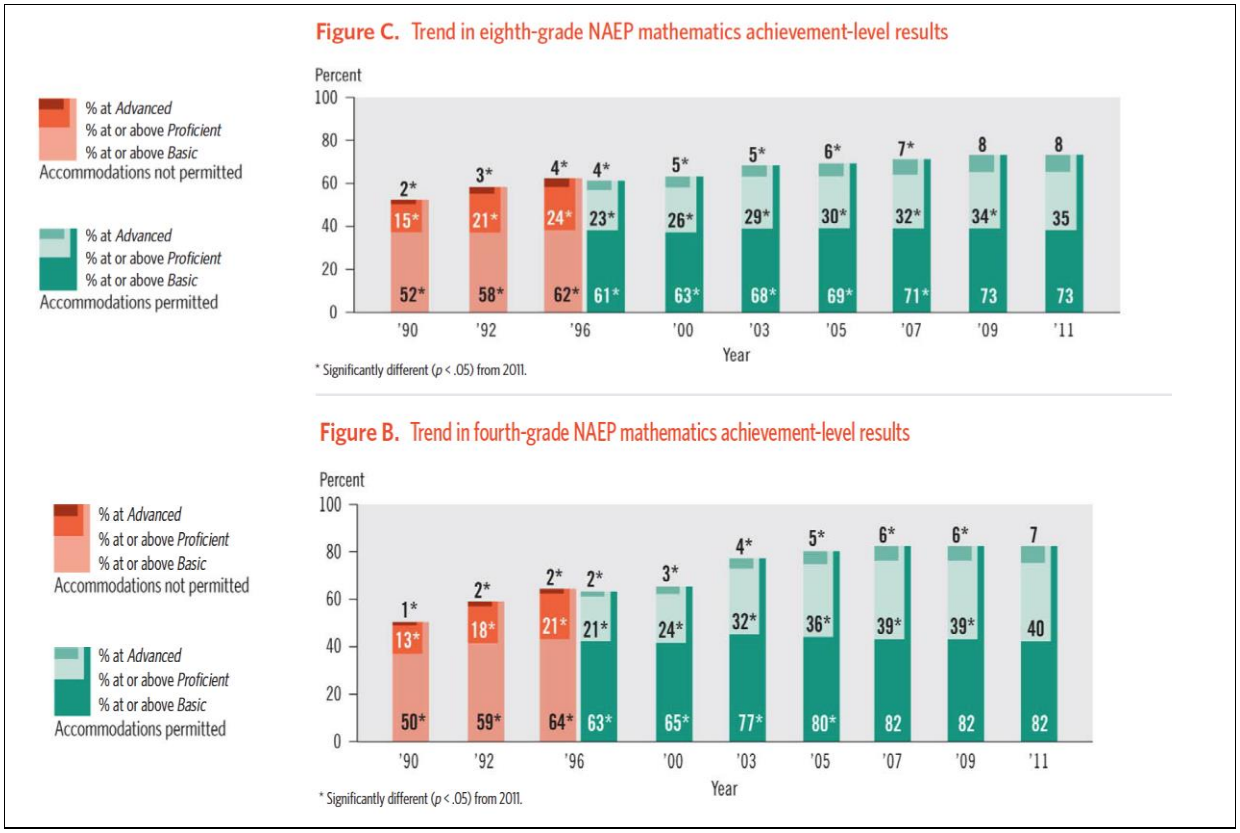
Figure 2. Trend in 4th and 8th grade NAEP mathematics average scores, 1990-2009



Source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, NAEP, various years, 1990-2009 Mathematics Assessments, at https://nces.ed.gov/nationsreportcard/pubs/main2009/2010451.aspx .

Another way of comparing data over the years is to examine the percentages of students at various achievement levels. In Figure 3, we see trends that parallel the trends in the mean scores. At 4th grade, only 13% of students were deemed proficient or advanced in 1990; that had increased to 40% in 2011. At 8th grade, 15% of students were deemed proficient or advanced in 1990; that had increased to 35% in 2011.

Figure 3. NAEP National Achievement Level Percents, Grades 4 and 8, 1990-2011



Source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, NAEP, various years, 1990-2011 Mathematics Assessments, at https://nces.ed.gov/nationsreportcard/pubs/main2011/2012458.aspx .

However, I think that these improvement trends from 1990 to 2009 were not viewed as significant by the people who were the main conceptualizers of the Common Core because, for the most part, the conceptualizers were college-level mathematicians or people influenced by those mathematicians.

They were bothered by two long-standing problems not addressed by scores on National Assessment: first, the great numbers of college students who are required to take remedial mathematics either because they have not taken enough mathematics in high school, or even when they have on paper taken enough mathematics. By this, we are speaking of college students who need to take the equivalent of second-year algebra or below because they have not learned this material well-enough to place out of such courses on a placement exam, or students who place into a precalculus course or lower even though they have taken a calculus course as 12th graders.

Over the years the lack of background for college-level mathematics has become an increasingly prevalant problem because more and more disciplines require significant mathematics, more and more colleges require passing a level of mathematics in order to receive a degree, and the high cost of college means that having to take courses over mathematics that *could* or *should have been learned in high school* may keep some students from ever obtaining a college degree.

CBMS Enrollment Data

Most remedial courses take place in two-year colleges. In the fall of 2010, *at two-year colleges*, where about a quarter of high school graduates enrolled immediately after graduation, about 57% of all enrollments in mathematics courses (including statistics courses) – over 1,150,000 students - were in what are labelled by the CBMS studies as "pre-college mathematics" courses, covering mathematics that is typically taught before grde 12: arithmetic, pre-algebra, elementary and intermediate algebra, and geometry). I've labelled these in red in Table 2. That percentage – 57% - had not changed since 1990. [Source: 2010 CBMS Survey of Undergraduate Programs, p. 136.] But in the five years from 2010 to 2015, the percentage of enrollment in these courses fell to 41%, primarily due to decreases in enrollments in arithmetic, pre-algebra, and elementary algebra, and increases in enrollments in college algebra and elementary statistics. [Source: 2015 CBMS Survey of Undergraduate Programs, p. 164.] This can be viewed as a significant improvement but it is hard to pin the cause to Common Core because so much of a two-year college student's basic mathematics background in 2015 would have occurred prior to 2012 and would not have been affected by the Common Core movement.

Table 2: Enrollments (000s) in mathematics and statistics in mathematics programs in 2-year colleges 2000-2015

Course	2000	2005	2010	2015
Arith. and Basic Math	122	104	146	71
Pre-algebra	87	137	226	127
Elementary Algebra	292	380	428	277
Intermediate Algebra	255	336	344	299
Geometry	7	7	6	8
All pre-calculus	274	321	368	445

All calculus	106	107	137	152
All other math	222	317	368	537
TOTAL*	1347	1696	2024	1918

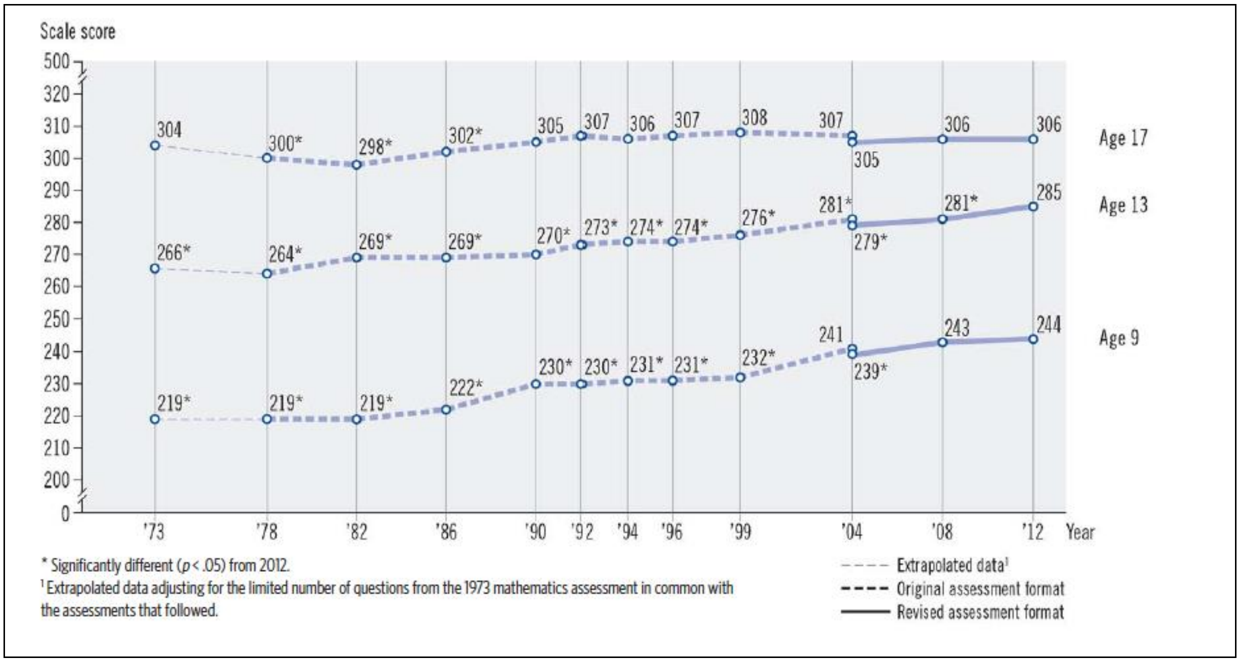
*Round-off from values used to create the values may make column sums seem inaccurate.
Source: 2015 CBMS Survey of Undergraduate Mathematics Programs

Also, as you can see, the enrollments tend to be volatile over the years, indicating that other factors are at play here. For instance, statistics, a part of "all other math" in this table, enrolled 71,000 students in 2000 and 251,000 students in 2015. The next survey of enrollments will be in the fall of 2020. It will be interesting to see the changes in the data but it will probably be difficult to attach any changes to effects of the Common Core.

The second problem bothering college-level mathematicians was the fact that although the United States was certainly in the top rank of countries producing high-level mathematicians, and maybe even #1 in the world, this was not reflected in PISA and TIMSS results, and a significant percentage of graduate students in mathematics came from other countries. So, from an international perspective, it seems that we could be doing much better.

It is easy to argue that the reason that enrollments in remedial mathematics at the college level remain high is because more majors and more and more colleges require more mathematics than they did two or three decades ago. But the data backing the need to establish detailed guidelines stretching through *high school* mathematics was from the NAEP long-term longitudinal study that dates back to 1973 (Figure 4

Figure 4: Trend in NAEP mathematics average scores for 9-, 13, and 17-year-old students, 1973-2012



Source: *The Nation's Report Card: Trends in Academic Progress 2012*, at https://nces.ed.gov/nationsreportcard/pubs/main2012/2013456.aspx .

We have data only through 2012 because the last time this study was undertaken was then. The results of this study, done with 9-, 13-, and 17-year-olds rather than at grades 4, 8, and 12, mimic the increases of the two younger groups over the years, including from 1999 through 2008. The 9-year-olds mean score increased from 219 in 1973 to 232 in 1999 and 243 in 2008; the 13-year-olds increased from 266 in 1973 to 276 in 1999 and to 281 in 2008. But the mean score for 17-year-olds was remarkably constant, 304 in 1973, 308 in 1999, and 305 in 2008. It certainly seems to be an anomaly that mean scores would increase for the younger students without a corresponding increase at the high school level. [24.5 minutes]

But there is another way to look at the 12th-grade data. This way is found in the report on the NAEP longitudinal studies in mathematics and reading published in 2013.

Table 3. NAEP longitudinal study of 17-year-olds in mathematics average scores in 1973 and 2012

Group	1973 average	2012 average	Change	% of sample in 1973	% of sample In 2012
White	310	314	**+4**	83	56
Black	270	288	**+18**	12	14
Hispanic	277	294	**+17**	4	22
Other*				1	8

All students	**304**	**306**	**+2 (not sig. diff)**	**100**	**100**

*Asian, Pacific Islander, Native American; change not calculated because so few in 1973.

Source: *NAEP 2012 Trends in Academic Progress: Reading 1971-2012, Mathematics 1973-2012*, and other NAEP reports.

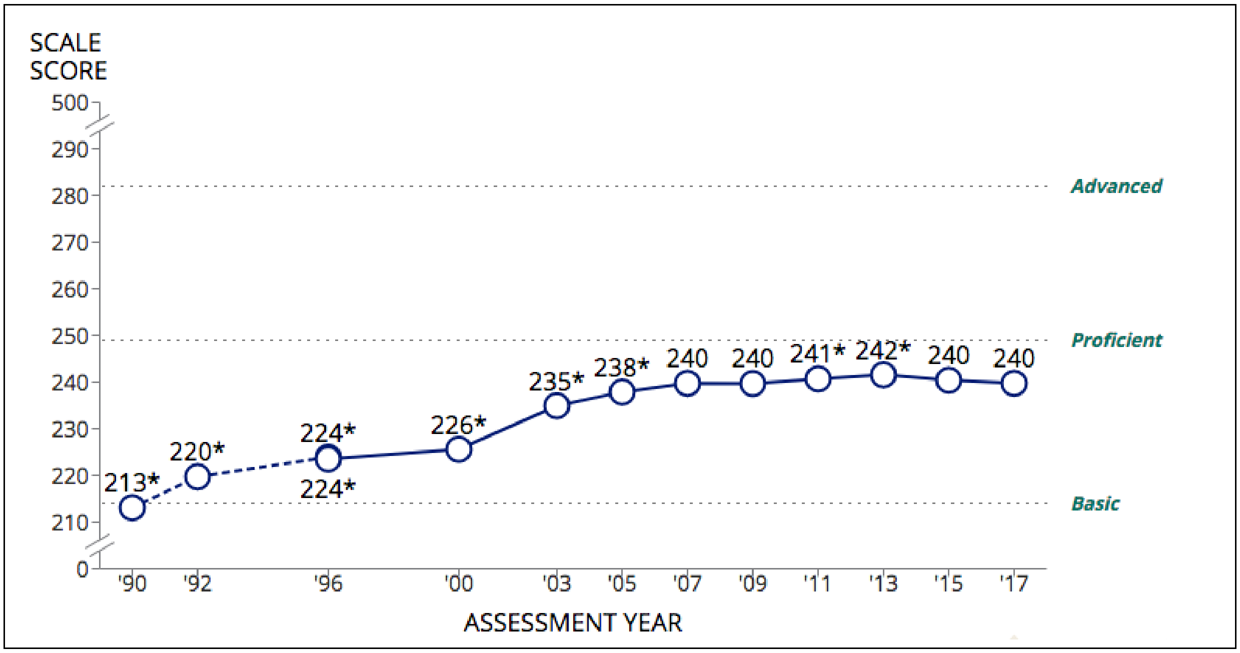
Describing this table, the NAEP report notes (p. 41): "White-Black and White-Hispanic gaps narrowed at age 17 because Black and Hispanic students made larger gains from 1973 than White students. Average mathematics scores for 17-year-olds increased 4 points from the first assessment year for White students, 18 points for Black students, and 17 points for Hispanic students. The changing makeup of the student population is one reason why the overall average score for 17-year-olds has not changed significantly even though student groups within the overall population are making gains. When an increase in the proportion of typically lower performing students is accompanied by a decrease in the proportion of higher performing students, the overall average score can remain unchanged even though the average scores for both higher and lower performing groups increase. This phenomenon is known as Simpson's paradox."

I do not show it here, but the increase in the mean score of Black students occurred almost entirely between 1973 and 1990; the increase for Hispanic students occurred almost entirely between 1973 and 1992. Seventeen-year-old students scores increased for all groups by only 2 points in the 20 years between 1992 and 2012 (NAEP, Ibid, pp. 39-40). SAT and ACT scores increased more in those years, and so did the numbers of students obtaining scores of 3 or higher on the AP calculus tests, but it is reasonable to say that at the time the Common Core standards were written, the NAEP high school data did not consistently reflect the increases that were found at elementary and middle school levels.

NAEP Data Since the Common Core Appeared

Now let us add to these data the regular NAEP tests given since 2009. Since 2003, the NAEP tests have been given every two years at Grades 4 and 8. Figure 5 shows the average national score for each administration until 2017. The average for recent years is constant, at 240.

Figure 5: NAEP 4^{th} grade mathematics average scores 1990-2017



Source: https://www.nationsreportcard.gov/math_2017/nation/scores/?grade=4

But again there is a manifestation of Simpson's Paradox (Table 4). Scores for Blacks, Hispanics, and Native Americans have increased slightly while scores for Asian Americans have increased significantly. It is the change in the composition of the sample from White to Hispanic or multi-ethnic that has caused the mean to remain the same.

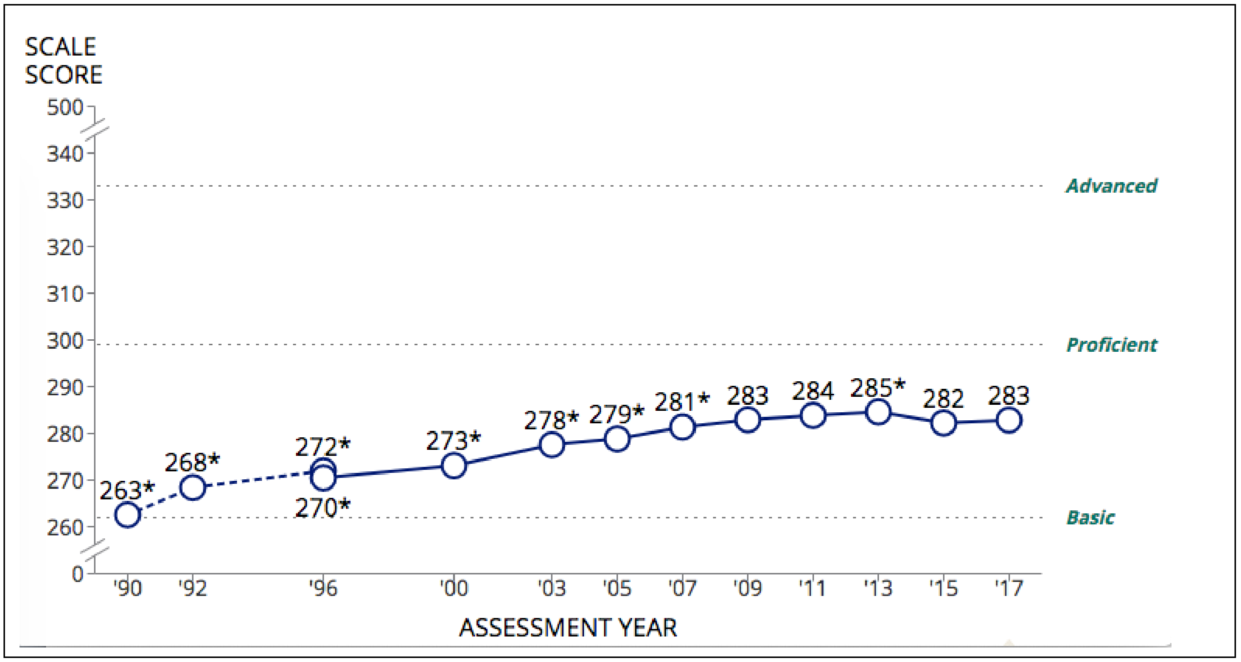
Table 4: NAEP average scores of 4th graders in mathematics 2009 and 2017, disaggregated

Group	2009 mean	2017 mean	Change	% of sample in 2009	% of sample In 2017
White	248	248	0	56	48
Black	222	223	+1	16	15
Hispanic	227	229	+2	21	26
Asian/P.I.	255	260	+5	5	5
Multi	---	245		---	4
NativeAmer	225	227	+2	1	1
All students	**240**	**240**	**0**	**100**	**100**

Source: NAEP Reports from 2009 and 2017

Eighth grade shows the same pattern as 4th grade (Figure 6). The national score average was 283 in 2009 and again 283 in 2017, no change. But scores for Hispanics increased and scores for Asians increased quite a bit.

Figure 6: NAEP 8th grade mathematics average scores 1990-2017



Source: https://www.nationsreportcard.gov/math_2017/nation/scores/?grade=8

Again it is the change in the percents of the sample that were white, Hispanic, and multi-ethnic that has caused the mean to remain the same despite more increases than decreases in the subgroups (Table 5).

Table 5: NAEP average scores of 8th graders in mathematics 2009 and 2017, disaggregated

Group	2009 mean	2017 mean	Change	% of sample in 2009	% of sample In 2017
White	293	293	0	58	51
Black	261	260	-1	15	14
Hispanic	266	269	+3	20	24
Asian/P.I.	301	310	+9	5	6
Multi	---	287		---	4
NativeAmer	266	267	+1	1	1
All students	**283**	**283**	**0**	**100**	**100**

Source: NAEP Reports from 2009 and 2017

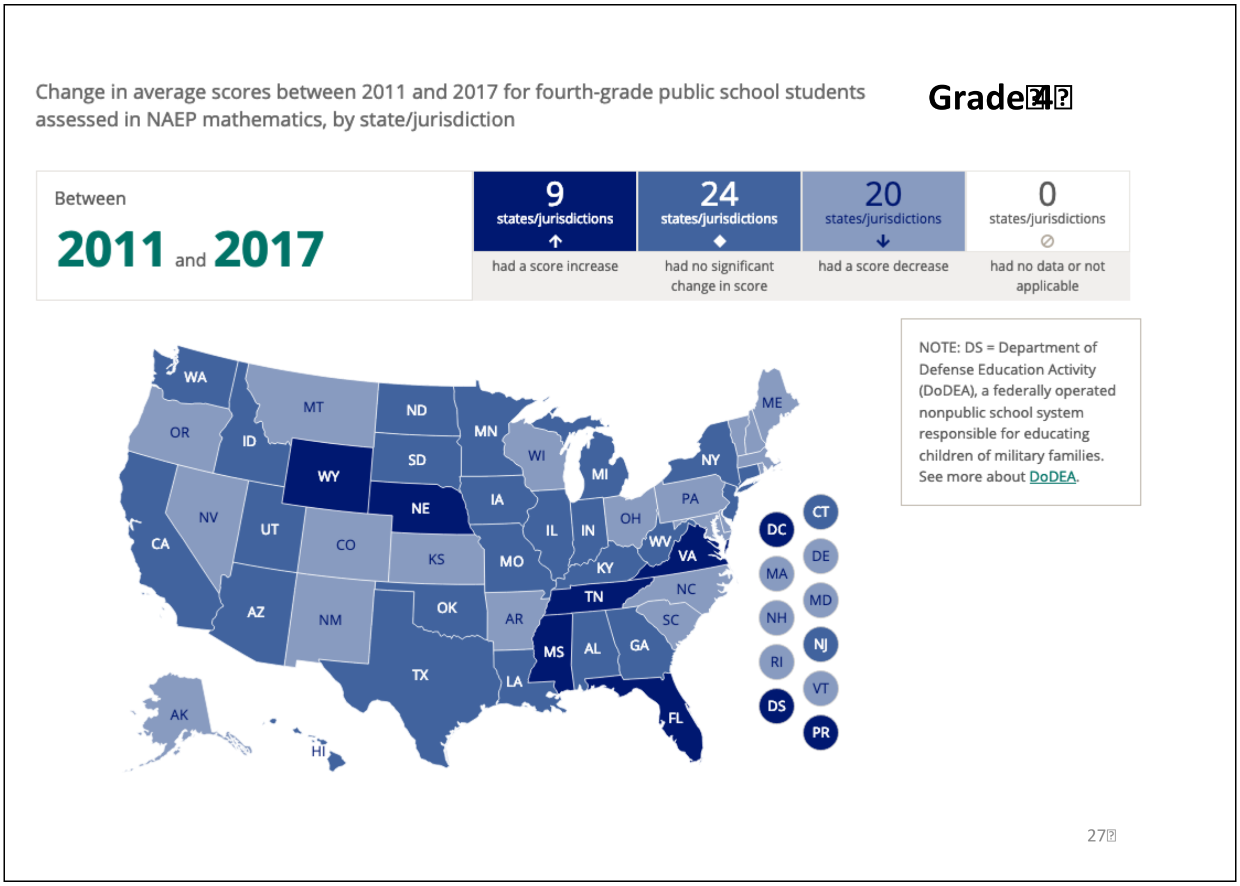
The NAEP test framework for grades 4 and 8 essentially dates from 1990, so it is different from the Common Core framework used for recent state tests. Is the difference enough to be significant? 10% of the Grade 4 NAEP is on statistics and probability; there is almost no statistics or probability in the Common Core before grade 5. But the NAEP framework was quite forward-looking, and includes a good amount of algebra at Grade 8 as well as geometric transformations in middle school and high school.

CAUTION: The partition of the national sample into these subgroups is somewhat controversial because the low scores of Blacks and Hispanics are translated by some of the public into the incorrect view that students in these groups just aren't as capable as in the other groups, and that publicizing these subscores can become, in some schools and classrooms, a self-fulfilling prophecy. But for policy makers, and for analyses like the one I am giving, these subgroup scores are interesting and can be useful in allocating resources for improving mathematics performance.

Looking at NAEP scores for the entire nation can suggest to us whether the Common Core idea was a good one even if some states did not follow the Common Core. After all, it might be that the Common Core idea was good for some states and not good for others, and each state knew best which is the better route for them to take.

Figure 7 exhibits the degree of changes of scores in all 50 states and DC, Puerto Rico, and the Department of Defense schools from 2011 to 2017 at Grade 4. NAEP calls these non-state test areas jurisdications. Only 9 of the 53 states/jurisdictions had higher average scores in 2017 than in 2011 – they are colored in dark blue and include two of the five non-Common Core states. Twenty states/jurisdictions had lower scores in 2017 than in 2011; only Alaska among those 20 was a non-Common Core state.

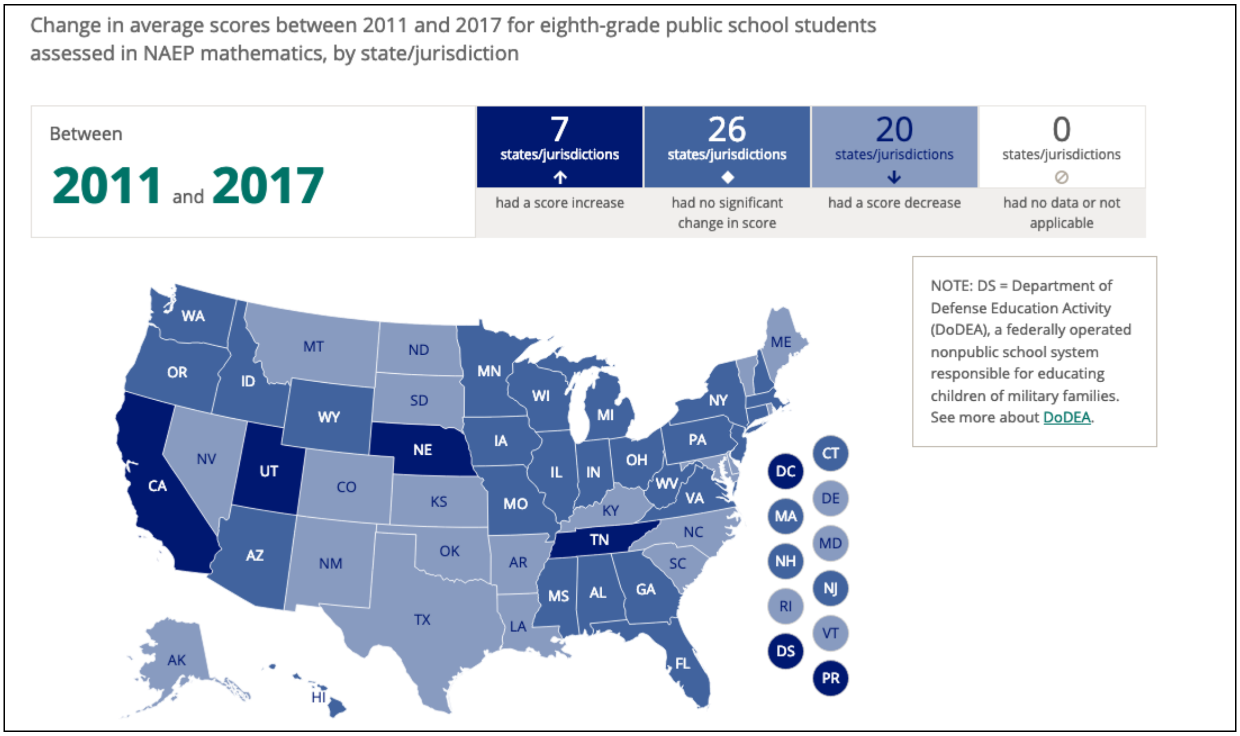
Figure 7: Change in average scores between 2011 and 2017 for 4^{th}-grade public school students assessed by NAEP



Source: NAEP Mathematics Report Card State Average Score Comparisons at https://www.nationsreportcard.gov/math_2017/states/scores?grade=4

At grade 8, 7 states/jurisdictions had an average score that increased from 2011 to 2017 (Figure 8). One of these is Nebraska, a non-Common Core state. But again many more states saw a decline than an increase, and three non-Common Core States were among these: Texas Oklahoma, and Alaska.

Figure 8: Change in average scores between 2011 and 2017 for 8th-grade public school students assessed by NAEP



Source: NAEP Mathematics Report Card State Average Score Comparisons at https://www.nationsreportcard.gov/math_2017/states/scores?grade=8

The framework for the 12th-grade NAEP was changed for the 2005 assessment so significantly that there exists only trends from 2005 on. And the 12th-grade NAEP was only administered in 2009, 2013, and 2015 after that. So we have nothing for 12th-graders more recent than 2015, and the 2015 results should not be viewed as a test of the Common Core because it is too early. Still, let us take a look.

Table 6: NAEP average scores of 12th graders in mathematics 2009 and 2015, disaggregated

Group	2009 mean	2015 mean	Change	% of sample in 2009	% of sample In 2015
White	161	160	-1	59	55
Black	131	130	-1	16	14
Hispanic	138	139	+1	19	22
Asian/P.I.	175	170	-5	6	6
Multi	158	157	-1	---	---
NativeAmer	144	138	-6	1	1

All Students	**153**	**152**	**-1**	**100**	**90**

Source: https://www.nationsreportcard.gov/reading_math_g12_2015/#

Table 6 shows that the average scores of 12th graders in 2015 were about the same as in 2009 in all groups, except that the mean score of Asian/Pacific Island peoples and of Native Americans were lower. I believe the 12th-grade NAEP has been administered earlier this year, so next year we will get the first results of 12th-graders on NAEP tests that can be expected to have been influence from the Common Core.

Altogether, we can say that after two decades of significant increases in NAEP average scores at grades 4 and 8, the NAEP scores over the past decade have stagnated or declined, and we do not yet have enough data at grade 12 to justify any conclusion.

SAT and ACT data

The two most prevalent tests given to high school seniors are the SAT and ACT. Unfortunately for this analysis, we cannot use national means on the SAT because beginning in March 2016 students took a redesigned SAT and the College Board has advised that we should not compare results before that date with results after that date. But even if we could have comparable SAT scores over the years, we would have difficulty because the sample changes from year to year.

The same difficulty plagues the ACT. Indeed, the number of students taking the ACT almost doubled between 1990 and 2010, and it has increased by another 20% since then. There are now 17 states in which all graduating seniors take the ACT; in only 3 states do all graduates take the SAT. Still, it is instructive to look at average math scores on the ACT over time, particularly since news accounts in various media outlets pointed to national ACT scores for 2018 as the lowest in 20 years.

To equate the sample from year to year, I decided looked at ACT data in a slightly different way than I've seen anyone else look (Table 7). This table allows a comparison of the average math scores over time with those in English and equates the sample for each year because the same students took both of these parts of the ACT.

Table 7: ACT Math and English average scores 1990-2018

Year	Participants (thousands)	Math Average	English Average	Math - English
1990	817	19.9	20.5	-0.6
1995	945	20.2	20.2	0
2000	1065	20.7	20.5	0.2
2005	1186	20.7	20.4	0.3
2010	**1569**	**21.0**	**20.5**	**0.5**

2015	1924	20.8	20.4	0.4
2016	2090	20.6	20.1	0.5
2017	2030	20.7	20.3	0.4
2018	**1910**	**20.5**	**20.2**	**0.3**

Source (for 1995-2017): https://nces.ed.gov/fastfacts/display.asp?id=897; for 1990 and 2018, ACT reports for that year

When we do this equating, we see that, indeed from 1990 to 2010, the average math score not only increased, but increased at a time when the average English score stagnated. But if we compare recent years (shown with a pink background) with 2010, we see that the average math score has declined to its lowest level since before 2000. Though there has been a decline also in the English average score, the decline is not as great as the math decline.

This negative showing on the ACT in recent years backs up the view that the Common Core has not had a positive effect on overall high school mathematics performance. The only caveat I found is a report by a person associated with one of the test-prep companies that offers classes to review or cram for the ACT in which the company asserts that the ACT mathematics test has become harder enough over the years that the test-prep company had to modify what it covers in its review classes. But I think ACT would argue that the scores are calibrated to be comparable from year to year.

Moving from National Data to State Data

I first wanted to look at Kentucky because this was the first state to give tests over the Common Core and thus would provide the longest time line for these tests. I could find newspaper reports that the scores in Kentucky had gone down but I could not find any numbers. I gave up. Then, since so many other states have changed the tests they have given, I thought about looking only at states that have stuck with either PARCC or Smarter Balance from the beginning.

But so many states have left PARCC that there will be almost no longitudinal data after this year in those states. And, in my state of Illinois, a state that has just left PARCC, one school superintendent applauded the decision to leave PARCC noting that we needed a test that was more closely aligned with the Common Core! More closely aligned? Surely PARCC was designed to be very closely aligned, but if the public perception is that it is not, then given the brief amount of time for this analysis, I chose not to consider PARCC states.

Here I show results from three Smarter Balanced states, Washington, Delaware, and California. The state of Washington is one of the higher-scoring states in the nation, California is one of the lower-scoring, and Delaware is somewhere in the middle, so they provide a nice trio to examine.

The state of Washington has stuck with Smarter Balanced (SBAC) and posts the frequency distribution of students having each score on the SBAC test. On this distribution they identify the SBAC cut scores that have not changed since the first administration of the test in 2015. These cut scores are the differences between what Smarter Balanced calls Levels 1, 2, 3, and 4. My opinion based on many years of working on cut scores is that people in our field often disagree on what constitutes prformance at a particular level such as basic or proficient or advanced. But the important aspect of these data is that the Smarter Balanced cut scores for a given grade have stayed the same since the first administration of the test, which enables a comparison of performance from year to year.

When a test is administered year after year, even when equivalent forms are substituted, you can expect that teachers will become more savvy as to what is on the test, students will also be more comfortable, and scores should go up a little from year to year. In Table 8, it means that a smaller percent of students should score at or below the cut scores as time goes on. Notice that if we look at Grade 3 we see that as time has gone on, more students have scored below the lowest cut score, but also more students have scored above the highest cut score. The pattern for Grade 4 is similar. The distribution seems more spread out at both grades, something that you would not expect if the same curriculum is being mandated for all students. But it is the middle cut score that may be the one we want to use as a measure of success for the Common Core. This gives the percent of students who did not meet the standards. Again we want and expect that percent to get lower as the years go on. But this percent basically stays the same for all four year 2015 through 2018 in both Grades 3 and 4.

Table 8: Washington State SBAC percent of students at or below cut scores at each grade 3 through 8, from 2015 through 2018

		Lowest cut score	**Middle cut score**	**Highest cut score**
Grade 3	**Year**	**2381**	**2436**	**2501**
	2015	20.2	42.9	74.8
	2016	19.0	40.7	72.3
	2017	20.6	41.9	71.8
	2018	21.2	41.9	71.3
Grade 4	**Year**	**2411**	**2485**	**2549**
	2015	16.4	45.7	74.9
	2016	16.4	44.5	73.3
	2017	18.0	45.5	73.7
	2018	18.6	45.8	73.6
Grade 5	**Year**	**2455**	**2528**	**2579**

	2015	24.2	51.6	72.6
	2016	24.1	50.6	71.2
	2017	25.8	51.2	71.1
	2018	25.8	51.0	70.9
Grade 6	**Year**	**2473**	**2552**	**2610**
	2015	24.7	54.1	76.5
	2016	24.3	51.6	73.6
	2017	24.6	51.6	73.9
	2018	24.6	51.3	73.3
Grade 7	**Year**	**2484**	**2567**	**2635**
	2015	23.8	51.2	75.9
	2016	22.7	49.6	74.2
	2017	24.0	49.6	73.1
	2018	25.0	50.2	73.3
Grade 8	**Year**	**2504**	**2586**	**2653**
	2015	27.8	52.6	73.3
	2016	27.0	51.1	72.0
	2017	28.2	51.5	71.8
	2018	28.4	51.3	71.2

Source: Each year's Washington Summary Report Card at www.reportcard.ospi.k12.wa.us

At Grades 5 and 6, again over time more students are exceeding the highest cut score. It may be only a few percent more in 2018 than 2015, but to expect more would be to expect too much. However, the middle cut score, the percent of students who did not meet the standards, has essentially remained the same in Grade 5 and since 2016 has not changed in Grade 6.

The cut scores at Grades 7 and 8 tend to follow a pattern that shows the greatest improvement at all grade levels was between 2015 and 2016, and again it looks as if, over time, more students are scoring higher than the highest cut score, and more students are scoring at or below the lowest cut score, while the percent of students who met or exceeded the standards has not changed since 2016..

I would show similar data for Grade 11 from Washington, but in 2018 the reporting was changed from "Grade 11" to "high school" and so I got the impression that the test could be taken by students not in grade 11 and so trends were lost.

The same Smarter Balanced test given in Washington is also given in California. But the results have not been reported in the same way as in Washington. To understand the

California data, examine the bottom row of the Washington results in Table 8. The three cut scores split the Grade 8 2018 population into 4 parts, a lowest part of 28.4%, a next part of 22.9% - that is the difference between 51.3 and 28.4 – a third part of 19.9%, and a 4th part of 28.8%, the percent of students that scored higher than the highest cut score.

It is with these four percents that California reports its Smarter Balanced scores. It calls these four sections: Standards Not Met, Standards Nearly Met, Standards Met, and Standards Exceeded. For Washington, Table 8 shows, the four sections divide the population reasonably equally into about 25% that do not meet standards, another 25% that nearly meet them, and about 50% who meet or exceed the standards.

Table 9: California SBAC percent of students *who did not meet or nearly meet standards* in grades 3 through 8 and grade 11, from 2015 through 2018

Grade	2015	2016	2017	2018
3	33%	29%	28%	28%
4	31%	28%	28%	26%
5	41%	39%	39%	37%
6	36%	35%	35%	35%
7	37%	34%	36%	37%
8	41%	39%	40%	40%
11	45%	43%	44%	46%

Source: California Assessment of Student Performance and Progress Test Results at https://caaspp.cde.ca.gov

In California, as seen in Table 9, about 10% more students do not meet or nearly meet the standards than in Washington. Over time, the performance is getting better in grades 3 through 5. But in grades 6-8 and 11 there has been no significant change in the number of students not meeting standards.

Table 10: California SBAC percent of students *who met or exceeded standards* in grades 3 through 8 and grade 11, from 2015 through 2018

Grade	2015	2016	2017	2018
3	40%	46%	47%	49%
4	35%	38%	40%	43%
5	30%	33%	34%	36%

6	33%	35%	36%	38%
7	34%	36%	37%	37%
8	33%	36%	36%	37%
11	29%	33%	32%	31%

Source: California Assessment of Student Performance and Progress Test Results at https://caaspp.cde.ca.gov

The situation is rosier with respect to the percents of California students *meeting or exceeding* the Smarter Balanced Standard cutoffs (Table 10). In all grades 3 through 8, the percent has increased significantly since 2015. This is the most positive result I found in my look at test performance since the CCSSM were instituted. Grade 11 seems to have stagnated. Again these percents are about 10% lower than the corresponding percents in Washington state. They parallel the differences in performance found in Washington and California students on NAEP at grades 4 and 8.

Delaware's SBAC scores in grades 3-8 show a great improvement from 2015 to 2016 but little or no improvement since then (Table 11). Delaware used to, but does not now use SBAC for its high school tests.

Table 11: Delaware SBAC percent of students *who met or exceeded standards* in grades 3 through 8, from 2015 through 2018

Grade	2015	2016	2017	2018
3	53%	55%	54%	54%
4	47%	50%	50%	50%
5	38%	42%	44%	43%
6	34%	37%	40%	40%
7	37%	40%	42%	39%
8	35%	38%	38%	39%

Source: Delaware Department of Education yearly Smarter Balanced State Summaries
Note: This table is different from the one presented in the NCSM, NCTM, and USACAS talks on which this paper is based, but the prose remains.

Altogether, since 2015 there has been a small increase in SBAC scores from the states I examined, but most between 2015 and 2016. That increase is certainly influenced by familiarity with the tests but some of the increase might be attributable to the Common Core itself.

Advanced Placement Data

So far we have seen a lot of performance data from grades K-12, but no performance at the college level. This is because I could find no broad studies involving college level-students. But we do have good long-term data on enrollment and performance for Advanced Placement tests.

Table 12: Number of students taking AP Calculus tests (AB or BC), 1990-2018

Year	Number	Year	Number	Year	Number
1990	77,634	2000	171,418	2010	324,865
1991	85,708	2001	184,905	2011	340,551
1992	93,196	2002	199,309	2012	361,397
1993	101,945	2003	212,794	2013	387,297
1994	110,508	2004	225,228	2014	406,185
1995	122,613	2005	240,407	2015	421,239
1996	126,558	2006	255,784	2016	433,146
1997	134,502	2007	276,004	2017	448,613
1998	144,759	2008	291,938	2018	447,914
1999	158,468	2009	303,553		

Source: Program Summary Reports from AP Reports to the Nation 1990-2018

The numbers of students taking calculus in high school consistently rose from 1990 to 2017 (Table 12), so much so that since 2005 more students have taken calculus in high school than in college. Most calculus enrollees in high school take the AP tests, so AP enrollments have also consistently risen, except for this past year. In 1990 only about 2% of the age cohort took the AP calculus tests. By 2000, that percent had doubled to about 4%. By 2010 that percent had almost doubled again, to 8%, and in 2018 it was about 11% of the age cohort or 13% of all high school graduates. The % increase from year to year is decreasing but this is because the roughly 7% increase per year from 1990 to 2010 could never be sustained.

But from 2017 to 2018 there was a little decrease, less than 1%, a decrease that may not be significant because it is overcompensated by increases in AP statistics enrollment. AP statistics began relatively recently, in 1997 with less than 8,000 students. Last year, in 2018, over 5% of the age cohort took the AP statistics test, 222,000 students. This is about half as many students as took the AB or BC calculus tests.

The huge increase in the numbers of students taking the AP calculus tests has occurred with little decrease in the percent of students scoring 4 or 5 on the AB or 3, 4, or 5 on the BC exam. I call these students "successful" because they will likely get some college calculus credit for scores that high (Table 13).

Table 13: Succssful AP Calculus atudents 1999-2018

Year	4 on AB	5 on AB	3 onBC	4 on BC	5 on BC	Total
1999	27,936	19,763	6,797	5,566	11,616	**71,678**
2004	34,854	35,755	10,496	9,416	19,971	110,492
2010	40,418	52,148	14,218	12,164	39,012	**157,960**
2012	45,523	67,394	14,957	15,231	47,553	190,658
2015	51,518	66,045	21,338	19,431	53,836	212,168
2016	53,467	76,496	21,441	19,191	60,632	**231,227**
2017	56,775	59,250	26,341	23,987	56,422	**222,775**
2018	53,255	59,733	28,891	25,982	56,324	**224,185**

In 1999, 38% of AB test-takers scored 4 or 5; 79% of BC test-takers scored 3, 4, or 5.
In 2010, 39% of AB test-takers scored 4 or 5; 83% of BC test-takers scored 3, 4, or 5.
In 2018, 37% of AB test-takers scored 4 or 5, 81% of BC test-takers scored 3, 4, or 5.
Source: Various AP Annual Reports to the Nation.

Since 2016 there has been a slight decline in the total number of successful AP calculus students. The Common Core recommends that preparation for an AP course not be done before grade 7, a sensible recommendation, but that it be done in most places by collapsing the curricula for grades 7, 8, and 9 into two years. Maybe this recommendation has caused schools to offer the AP calculus opportunity to fewer students.

International Comparisons: TIMMS and PISA

A goal of the Common Core, again represented by quotes in the original statement of objectives, was to bring performance up to that of the highest-scoring countries on PISA (Programme for International Student Assessment) and TIMSS (The International Mathematics and Science Study). This is an unrealistic goal for many reasons, but it still is helpful to see if, using these tests as a barometer, U.S. performance has improved in recent years.

Let us begin with PISA. PISA tests 15-year-olds around the world – over 70 nations in 2015 – in science, reading, and mathematics literacy every 3 years, and in each administration one of these areas is chosen for heavy emphasis. Consequently, it is only in every 9 years that mathematical content and mathematical processes are covered. They were last covered in 2012 and will next be covered in 2021. Consequently, we do not have enough data from

PISA to see any possible effects of the Common Core except on mathematics literacy. And even that data is only from 2015 because the 2018 results are not due to be announced until late this year.

Table 14: U.S. average scores on PISA literacy assessments

Year Subject	2000	2003	2006	2009	2012	2015
Science Literacy			489	502	497	496
Reading Literacy	504	495		500	498	497
Mathematics literacy		483	474	487	481	470

Source: https://nces.ed.gov/surveys/pisa/pisa2015/pisa2015highlights_6.asp

Table 14 shows the average U.S. scores on science, reading, and mathematics literacy. For PISA, *mathematics literacy* means the ability "to formulate, employ, and interpret mathematics in a variety of contexts to describe, predict, and explain phenomena, recognizing the role that mathematics plays in the world." The U.S. average mathematics literacy score in 2015 was substantially lower than the OECD average of 490. One state, Massachusetts, historically one of the states with the highest NAEP average scores, administered the PISA test as if it were a country, and its average score was 500. For comparison, 11 countries in Europe had an average score in 2015 higher than 500.

It is instructive to compare the U.S. average math literacy scores over time with those in Canada (Table 15). The difference is substantial.

Table 15: U.S. and Canada PISA Average Mathematics Literacy Scores Over Time

Year Country	2003	2006	2009	2012	2015
Canada	532	527	527	518	516
United States	483	474	487	481	470

Source: OECD PISA reports for the indicated years

Only two European countries, Switzerland and Estonia, had higher average scores than Canada, and they were only slightly higher. If we wish to improve mathematics literacy, we in the U.S. might look to our colleagues to the North to see what they are doing.

Now we turn to TIMSS, which has been offered at 4th grade and 8th grade every four years since 1995 except for 1999, when only 8th grade was offered. Fourteen constituents have participated in every TIMSS since 1995; I list their mean mathematics scores here. At 4th grade, the U.S. average is higher than that of the provinces of Quebec and Ontario, which appear in TIMSS as individual countries because they have substantially different curricula. Indeed, the U.S. looked pretty good at 4th grade in 2011 before the Common Core could have any effect. There was essentially no change between 2011 and 2015 so we will have to wait until the 2019 results are published to see if there will be any effect of the Common Core.

Table 16: Average 4th grade TIMSS math scores over time for all countries appearing in every administration since 1995, ordered by 2015 score

Country\Year	1995	2003	2007	2011	2015
Singapore	490	594	599	606	618
Hong Kong	537	575	607	602	615
Japan	567	565	568	585	593
England	484	531	541	542	546
United States	**518**	**518**	**529**	**541**	**539**
Canada(QUE)	550	506	519	533	536
Netherlands	549	540	535	540	530
Hungary	521	529	510	515	529
Slovenia	462	479	502	513	520
Australia	495	499	516	516	517
Canada(ONT)	489	511	512	518	512
Norway	476	451	473	495	493
New Zealand	469	493	492	486	491
Iran	387	389	402	431	431

Source: TIMSS reports for the indicated years

At 8th grade, the U.S. mean score is about the same as the countries of Western Europe, but still substantially lower than the countries of East Asia. This huge difference in performance between the U.S. and the counries of East Asia has been found ever since the first international study of mathematics achievement that was administered in the 1963-64 school year, over a half century ago. Again, we will have to wait until 2019 results are published to see any effects of the Common Core, but my guess is that any effects will be minimal.

Table 17: Average 8th grade TIMSS math scores over time for all countries appearing in every administration since 1995, ordered by 2015 score

Country\Year	1995	1999	2003	2007	2011	2015
Singapore	609	604	605	593	611	621

Korea	581	587	589	597	613	606
Hong Kong	569	582	586	572	586	594
Japan	581	579	570	570	570	586
Russia	524	526	508	512	539	538
England	498	496	498	513	507	518
United States	**492**	**502**	**504**	**508**	**509**	**518**
Hungary	527	532	529	517	505	514
Lithuania	472	482	502	506	502	512
Iran	418	422	411	403	415	436

Source: TIMSS reports for the indicated years

Results, Conclusions, and Interpretations of Test Scores

In this paper I have focused on one question: To what extent is the Common Core reaching its goal of improving the mathematics performance of students in the U.S.? I have given some achievement results and some conclusions from the data. Now I would like to give some summary interpretations.

The Common Core has provided explicit standards. We have materials that in large portions of the U.S. cannot be adopted unless they adhere strictly to the standards. We have teachers teaching to tests that are aligned to the standards. This is an unprecedent alignment of objectives, materials, and tests, unlike any we have ever seen in the United States, and we would expect to see growth - even from one year to the next – as the students taking the tests have had more years of the Common Core curriculum and as the teachers teaching the students have had more experience teaching that curriculum, and as both teachers and students would have more experience with the kinds of questions that appear on the Common Core tests.

With the caveat that even nine years in, we lack lots of data – particularly at Grade 12 and in colleges - the results we have would seem to be disappointing. Despite massive efforts in virtually every state to implement these standards, at grades 3-8 what improvement there has been is minimal and comes after two decades of significant improvement on all sorts of performance measures. On so many measures performance has stagnated, sometimes a very small increase, sometimes a decrease. So we must ask ourselves why we do not see consistent signs of substantial gain as was the case from 1990 to 2010.

I believe the major fault lies in that what was originally intended to be a core has instead become the entire curriculum. The Publishers Guidelines, enforced by the EdReports police, have taken out the ability of teachers to explore and play with concepts, to show directions in which a concept will lead, to extend skills, to move ahead when a class learns an idea. Publishers and teachers are also discouraged from reviewing content from previous years even when their students need that review. The alignment of tests with the curriculum means that teachers feel penalized if they insert their own interests into their teaching if those interests go beyond the Common Core curriculum. They teach to the test because they are discouraged or even downgraded if they go beyond the curriculum, or if they occasionally test over stuff from previous years.

In the back-to-basics era of the 1970s, reading scores plummeted, and I recall that one reason that was hypothesized is that the decline was a result of the trend to limit words in reading passages by their placement at an arbitrary grade level. This confirmed the view that you do *not* reach a particular grade level in reading by limiting the vocabulary to words at that level or lower. You must go beyond that level for context, to motivate your learning of new words and more sophisticated words, to have experience in working out meaning of words from their contexts. Indeed, new words pique curiosity and make reading more interesting.

So it is also with mathematics. It is appropriate to ask materials to cover all or virtually all of the objectives of the Common Core, including the standards for mathematical practice, but the advantages of that coverage are counterbalanced by essentially limiting the curriculum to those objectives. And covering important objectives only in one year may work for the best students, but for many (if not most) students at least two passes through important content are needed.

That is, the Common Core is not improving student performance because there has been a quite successful movement to limit the teaching only to the Common Core objectives, to hold students and classes back who might be ready to tackle more, to ignore students and classes who need to review concepts from earlier years, to discourage the exploration of extensions of the curriculum. The NCTM Standards and the resulting two decades of continual increases in test scores from 1990 to 2010 show that a strategy of giving latitude to states and school districts, and authors and publishers to develop a variety of approaches is, overall, a better approach. There may have been blunders in some places, but the benefits of freedom outweigh the losses due to the narrowing of what is taught.

We will not get substantial improvement until the Publishers Guidelines are either eliminated altogether or are modified to give freedom to teachers and to student materials, and until EdReports is taken off of its self-proclaimed perch as the sole judge of the quality of mathematics materials. We need to rethink the curriculum to make it more exciting so that learning mathematics has goals beyond test results or just preparing for the next year. To accomplish this, any new look at standards has to completely rethink the relationship between paper-and-pencil mathematics and technology that can do that mathematics.

It also may be that the CCSSM may not be working to improve core skills because today's core skills are not the same as they were before the days of 4-function, scientific, and graphing calculators, and CAS. It could be that a significant percent of students see no reason to learn paper-and-pencil algorithms in arithmetic, algebra and calculus in an age when all this can be done with technology. Surely some students, including some of our best students, feel that the standards on which they are being tested are entrenched in a paper-and-pencil world that is not in their time nor worth their time. Testing students on a computer is not the same as teaching students how to use their smartphone or tablet or laptop to perform tedious computations and to help in solving mathematics problems.

We should also take a good look at what causes 15-year-olds in so many countries to score much higher than 15-year-olds in the U.S. on mathematics literacy. That would seem to be a measure of preparation for *careers* that is a stated goal of the Common Core Standards but is not given enough attention in the standards themselves.

As for the more ambitious goal of catching up to the top-scoring countries in the world on international assessments, we will not catch up *simply* by having a curriculum like those countries; we already have a curriculum very much like those countries. The difference is that in those countries there is an expectation that *all* students will perform at the level of our best students.

A fundamental reason that the expectation of high performance of all students can be actuated by teachers in the high-performing countries of east Asia is that the elementary school teachers of mathematics themselves were among the best students in their classes when they were in school. In contrast, elementary school mathematics in the U.S. is typically taught by teachers who themselves were not top students in mathematics and who often carry the belief that there is some sort of "mathematics gene" that only some people have- and they are not among those so privileged. As long as that is their belief, we cannot expect those teachers to expect all of their students to perform at a high level, for they never had that expectation for themselves, and they do not even know what it means to perform at a high level.

We *do* have communities in the United States where the public schools perform as well as any of those in the high-performing countries. In those communities the salaries tend to be higher, so they can attract young college graduates who would otherwise go into higher-paying professions.

So either we have to have specialist mathematics teachers from grade 1 up, as is the case in Shanghai and some other places, or we need to raise the salaries of teachers high enough to attract our best students into teaching. In sum, even though I am someone who has worked in curriculum his entire life, and who believes in the power of a good curriculum, to bring U.S. perfomance even close to the top-scoring countries in the world will take more than a change in our curriculum. It will take a change in who goes into teaching and who stays, and for this we will need to improve the attractiveness of our profession by allowing more time to plan, reducing the number of hours facing students in a classroom, and raising salaries to compete with other professionals.

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